THE UNINTED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

FORM TWO NATIONAL ASSESSMET

0041 **BASIC MATHEMATICS**

Time: 2:30 Hours **ANSWERS** **Year: 2020.**

Instructions:

1. this paper consists of section A and B
2. Answer all questions
3. Each question carries Four marks.

1. (a) Write each of the numbers 18, 24, and 36 as a product of prime factors and hence find their greatest common factor.

Solution:
$18 = 2 \times 3 \times 3 = 2 \times 3^2$
$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$
$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

The common prime factors are 2 and 3.
For 2, the lowest power is 2^1.
For 3, the lowest power is 3^1.

Greatest Common Factor (GCF) = 2 x 3 = 6.

(b) Write the number 0.009765:
(i) Correct to three decimal places: 0.010
(ii) Correct to three significant figures: 0.00977

The place value of 9 in the given number is ten-thousandths.

2. (a) Find the value of the expression $5/2 - (3\frac{3}{5} \div 1\frac{1}{5} - 4/5)$.

Solution:
Step 1: Convert mixed numbers into improper fractions.
$3\frac{3}{5} = (5 \times 3 + 3)/5 = 18/5$
$1\frac{1}{5} = (5 \times 1 + 1)/5 = 6/5$
Step 2: Simplify the division operation.
$18/5 \div 6/5 = 18/5 \times 5/6 = 18/6 = 3$
Step 3: Simplify the expression.
The expression becomes:
5/2 - (3 - 4/5)
Step 4: Simplify inside the parentheses.
3 - 4/5 = (15/5 - 4/5) = 11/5
Step 5: Subtract.
5/2 - 11/5 = (25/10 - 22/10) = 3/10
Final answer: 3/10.

(b) (i) In a sales promotion, the price of a shirt costing shs. 15,000 is reduced by 15%. What is the new price of the shirt?

Solution:
Reduction = 15% of 15,000 = (15/100) x 15,000 = 2,250.
New price = 15,000 - 2,250 = 12,750.

(ii) Change 0.56 into a fraction in its simplest form.

Solution:
0.56 = 56/100 = 28/50 = 14/25.

Final simplified fraction: 14/25.

3. (a) A lorry carries 7.2 tonnes of sand from the mining area to the industrial site. On the way, 230 kg of sand either fall off or blow away. What mass of sand will remain by the end of the journey? Give the answer in tonnes.

Solution:
1 tonne = 1000 kg.
230 kg = 230 ÷ 1000 = 0.23 tonnes.

Remaining mass = 7.2 - 0.23 = 6.97 tonnes.

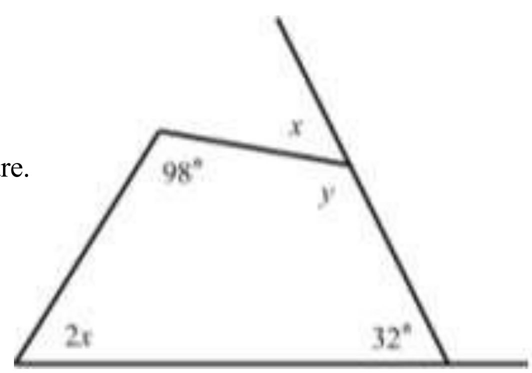
Final answer: 6.97 tonnes.

(b) An article was sold for shs 160,000 at a profit of 25%. Find the buying price of the article.

Solution:
Let the buying price be x.
Selling price = x + 25% of x = x(1 + 25/100) = 1.25x.
1.25x = 160,000.
x = 160,000 ÷ 1.25 = 128,000.

Final answer: 128,000 shs.

4. (a) Find the values of x and y in the following figure.



Solution:
In the triangle:
Angle sum property: x + y = 180°......................(i)
98 + 32 + 2x + y = 360° .... (ii)
Solving simultaneously,
x = 50°, y = 130°

(b) Suppose a metal wire is bent to form a semi-circle with a radius of 14 cm.
(i) Find the total length of the metal wire.

(ii) Find the area bounded by the metal wire.
Solution:
(i) Total length of the metal wire:
The total length includes the semi-circular arc and the diameter.
Circumference of a circle = $2\pi r$.
Length of the semi-circular arc = $(1/2) \times 2\pi r = \pi r$.
Diameter = $2r$.
Total length = $\pi r + 2r = \pi(14) + 2(14) = 44 + 28 = 72$ cm.

(ii) Area bounded by the metal wire:
Area of a semi-circle = $(1/2)\pi r^2$.
= $(1/2) \times \pi \times (14)^2$.
= $(1/2) \times \pi \times 196$.
= $98\pi \approx 98 \times 3.1416 = 307.88$ cm^2.

5. (a) The sum of two numbers is 127. If the difference between the numbers is 7, find the numbers.

Solution:
Let the two numbers be x and y.
$x + y = 127$.
$x - y = 7$.

Add the two equations:
$2x = 134$.
$x = 67$.
Substitute $x = 67$ into $x + y = 127$:
$67 + y = 127$.
$y = 127 - 67 = 60$.
Final answers: The numbers are 67 and 60.

(b) Solve the equation $x^2 - 10x + 13 = 0$ by completing the square. Leave the answer in surd form.
Solution:
Step 1: Rewrite the equation:
$x^2 - 10x = -13$.
Step 2: Complete the square.
Take half the coefficient of x, square it, and add to both sides:
$(10/2)^2 = 25$.
$x^2 - 10x + 25 = -13 + 25$.
$(x - 5)^2 = 12$.
Step 3: Solve for x.
$x - 5 = \pm\sqrt{12}$.
$x = 5 \pm \sqrt{12}$.
Final answers: $x = 5 + \sqrt{12}$, $x = 5 - \sqrt{12}$.

6. (a) (i) Find the equation of a line passing through the point P(-1, 4) and has a gradient of 10.
Solution:
The equation of a line is given by:
$y - y_1 = m(x - x_1)$, where m is the gradient and (x_1, y_1) is the point.

Substitute m = 10, $(x_1, y_1) = (-1, 4)$:
$y - 4 = 10(x + 1)$.
$y - 4 = 10x + 10$.
$y = 10x + 14$.
Final answer: $y = 10x + 14$.

6. (a) (ii) If the line of the equation obtained in part (a)(i) passes through the points (a, 0) and (0, b), find the values of a and b.

Solution:
The equation of the line is $y = 10x + 14$.

- When y = 0 (to find a):
$0 = 10a + 14$.
$10a = -14$.
$a = -14/10 = -1.4$.

- When x = 0 (to find b):
$b = 10(0) + 14 = 14$.

Final answers:
a = -1.4, b = 14.

(b) Find the image of the point P(4,1) when it is:
(i) Reflected in the x-axis:
Reflection in the x-axis changes y to -y.
Image = (4, -1).

(ii) Reflected in the line y = x:
Reflection in the line y = x swaps x and y.
Image = (1, 4).

(iii) Translated by the point T(3, 5):
Translation adds the components of T to the coordinates of P.
Image = (4 + 3, 1 + 5) = (7, 6).

Final answers:
(i) (4, -1).

(ii) (1, 4).
(iii) (7, 6).

7. (a) If $(3^{3+x})(5^{2-y}) = (1/3)^5(1/5)$, find the values of x and y.

Solution:
Rewrite the equation:
$(3^{3+x})(5^{2-y}) = 3^{-5} \times 5^{-1}$.

Equating the powers of 3:
$3^{3+x} = 3^{-5}$.
$3 + x = -5$.
$x = -5 - 3 = -8$.
Equating the powers of 5:
$5^{2-y} = 5^{-1}$.
$2 - y = -1$.
$y = 2 + 1 = 3$.
Final answers:
$x = -8$, $y = 3$.

(b) (i) Find the value of 0.0000234 × 120 in standard notation, correct to three significant figures.
Solution:
$0.0000234 = 2.34 \times 10^{-5}$.
$0.0000234 \times 120 = (2.34 \times 10^{-5}) \times (1.2 \times 10^{2})$.
$= 2.808 \times 10^{-3}$ (rounded to 3 significant figures).
Final answer: 2.81×10^{-3}.

(ii) Rationalize the denominator of the expression $\sqrt{5} / (\sqrt{3} + \sqrt{2})$.

Solution:
Multiply numerator and denominator by the conjugate of the denominator:
$(\sqrt{5} / (\sqrt{3} + \sqrt{2})) \times ((\sqrt{3} - \sqrt{2}) / (\sqrt{3} - \sqrt{2}))$
$= (\sqrt{5}(\sqrt{3} - \sqrt{2})) / ((\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}))$
$= (\sqrt{15} - \sqrt{10}) / (3 - 2)$
$= \sqrt{15} - \sqrt{10}$.
Final answer: $\sqrt{15} - \sqrt{10}$.

8. (a) In the following figure, AB = AC. Prove that ∠ABC and ∠ACD are also equal.

Solution:
Since AB = AC, the triangle ABC is isosceles.
In an isosceles triangle, the base angles are equal.

Thus, ∠ABC = ∠ACD.

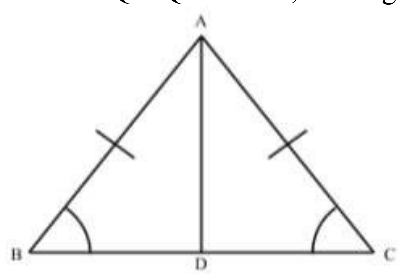
Final answer: Proven that ∠ABC = ∠ACD.

(b) If the rectangular metal sheets ABCD and WXYZ are similar, calculate the length of XY when AB = 2 cm, BC = 4 cm, and WX = 2.5 cm.

Solution:
The corresponding sides of similar rectangles are proportional.
AB/WX = BC/XY.
Substitute the values:
2/2.5 = 4/XY.
$2.5 \times 4 = 2 \times XY$.
10 = 2XY.
XY = 10/2 = 5 cm.

Final answer: XY = 5 cm.

9. (a) Figure PQR represents a triangular floor such that PQ = QR = 2 cm, and angle PQR is 90°. Find PR, correct to two decimal places.



Solution:
Using the Pythagorean theorem:
$PR^2 = PQ^2 + QR^2$.
$PR^2 = 2^2 + 2^2 = 4 + 4 = 8$.
$PR = \sqrt{8} = 2\sqrt{2} \approx 2.83$ cm.
Final answer: PR = 2.83 cm.

(b) Given that $\sin\theta = \sqrt{3}/2$ where θ is an acute angle; without using mathematical table, find:
(i) cosθ.
(ii) tanθ.

Solution:
(i) To find cosθ:
Using the Pythagorean identity:
$\sin^2\theta + \cos^2\theta = 1$.

Substitute $\sin\theta = \sqrt{3}/2$:
$(\sqrt{3}/2)^2 + \cos^2\theta = 1$.
$3/4 + \cos^2\theta = 1$.
$\cos^2\theta = 1 - 3/4 = 4/4 - 3/4 = 1/4$.
$\cos\theta = \pm\sqrt{(1/4)}$.
Since θ is acute, cosθ is positive:

$\cos\theta = 1/2$.
(ii) To find $\tan\theta$:
$\tan\theta = \sin\theta / \cos\theta$.
$\tan\theta = (\sqrt{3}/2) / (1/2)$.
$\tan\theta = \sqrt{3}$.

10. (a) In a certain village, 300 people were interviewed about their food preference. It was found that 200 people like banana, 120 people like rice, and 60 people like both banana and rice. By using a formula, find the number of people who like neither banana nor rice.

Solution:
Using the principle of inclusion and exclusion:
Let:
- Total population = 300.
- People who like banana = 200.
- People who like rice = 120.
- People who like both = 60.

The formula for the number of people who like either banana or rice:
$n(B \cup R) = n(B) + n(R) - n(B \cap R)$.
$n(B \cup R) = 200 + 120 - 60 = 260$.
The number of people who like neither banana nor rice:
Total population - $n(B \cup R) = 300 - 260 = 40$.

Final answer: 40 people.

(b) The masses of a group of students from Kilimani secondary school were recorded as shown in the following table:

Mass in kilograms	Frequency
31 – 40	2
41 – 50	5
51 – 60	3
61 – 70	9
71 – 80	1

(i) How many students are there in the group?
Solution:
Total number of students = Sum of frequencies = 2 + 5 + 3 + 9 + 1 = 20.

Final answer: 20 students.

(ii) State the class interval that has the largest number of students.
Solution:

The class interval with the largest frequency is 61 – 70 (frequency = 9).

Final answer: 61 – 70.

(iii) Prepare a table showing the class boundaries and the corresponding cumulative frequencies.

Solution:

To find class boundaries:

- Subtract 0.5 from the lower class limit.
- Add 0.5 to the upper class limit.

Class Interval	Class Boundaries	Frequency	Cumulative Frequency
31 – 40	30.5 – 40.5	2	2
41 – 50	40.5 – 50.5	5	7
51 – 60	50.5 – 60.5	3	10
61 – 70	60.5 – 70.5	9	19
71 – 80	70.5 – 80.5	1	20